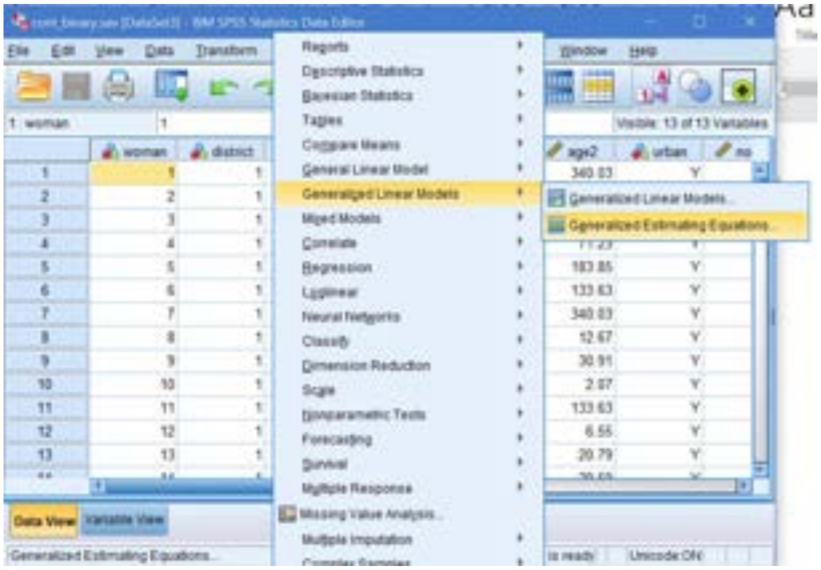
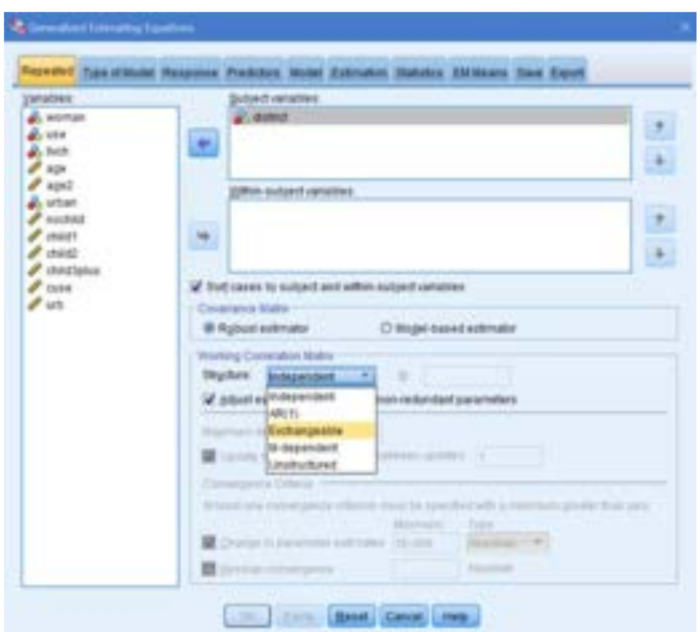
Appendix A: Estimating a generalized linear model (GLM) using GEEs with SPSS

The following guide shows how to use GEEs for a binary outcome using clustered data. The dataset is available at: http://francish.netlify.app/docs/cont_binary.sav.

1. With a dataset already open, select **Analyze → Generalized Linear Models → Generalized Estimating Equations**

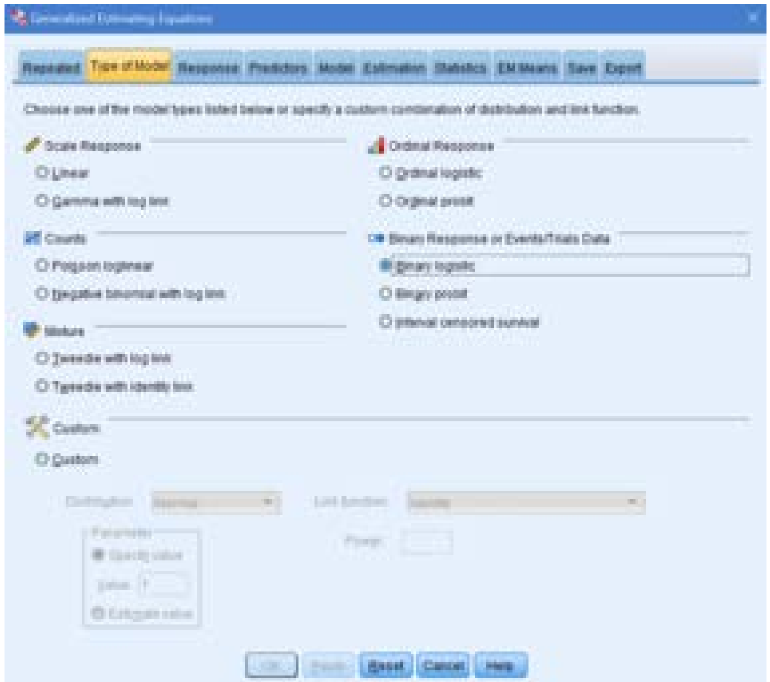


2. Under the **Repeated** tabs, select the clustering variable and place it in the **Subject variables** field. In this example, `district` is the clustering variable.
3. In the same window, choose the **Working Correlation Matrix** in the dropdown list which indicates **Structure**. As this example focuses on analyzing a cross-sectional clustered dataset, choose the **Exchangeable** option.

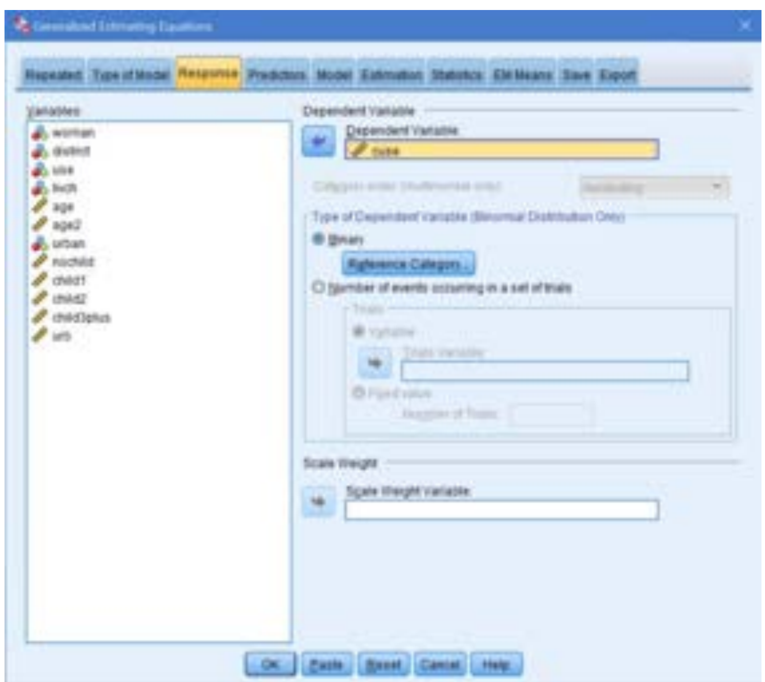


From Huang, F. L. (2021). Analyzing cross-sectionally clustered data using generalized estimating equations. *Journal of Educational and Behavioral Statistics.* doi: 10.3102/10769986211017480

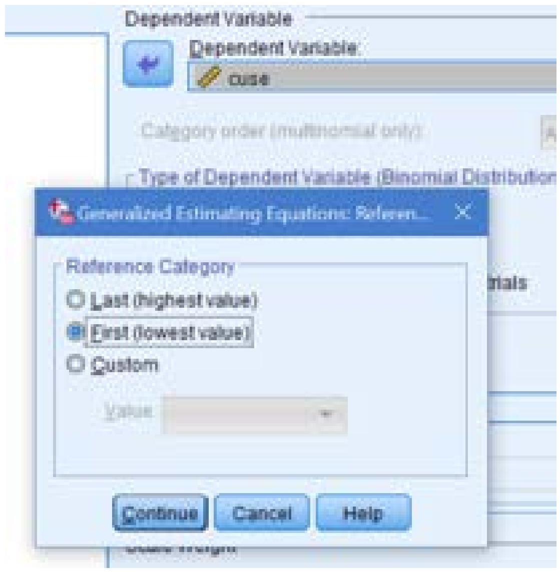
4. Click on the **Type of Model** tab. For continuous outcomes, no change is necessary on this screen. For this example, a logistic regression model will be run, select the **Binary logistic** option.



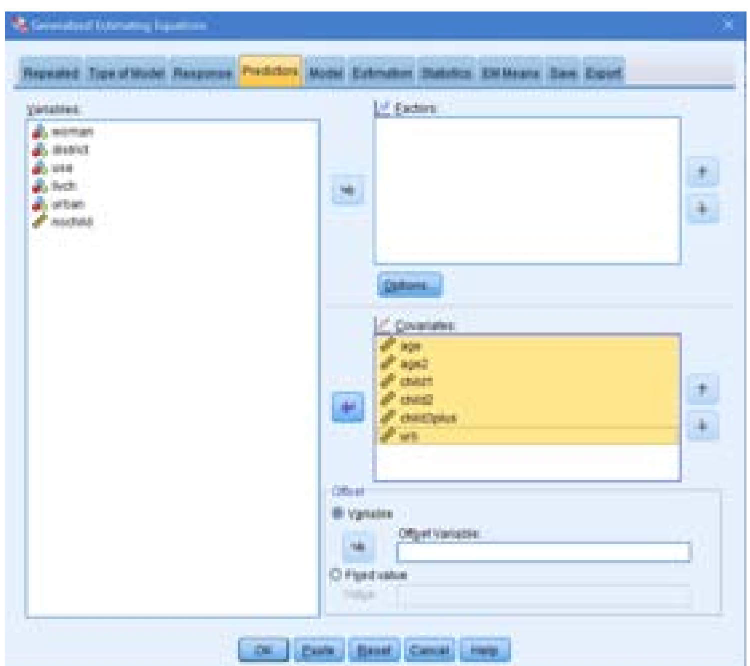
5. Click on the **Response** tab. Select the outcome variable (i.e., `cuse` in this example) and place it in the **Dependent Variable** field.

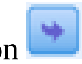


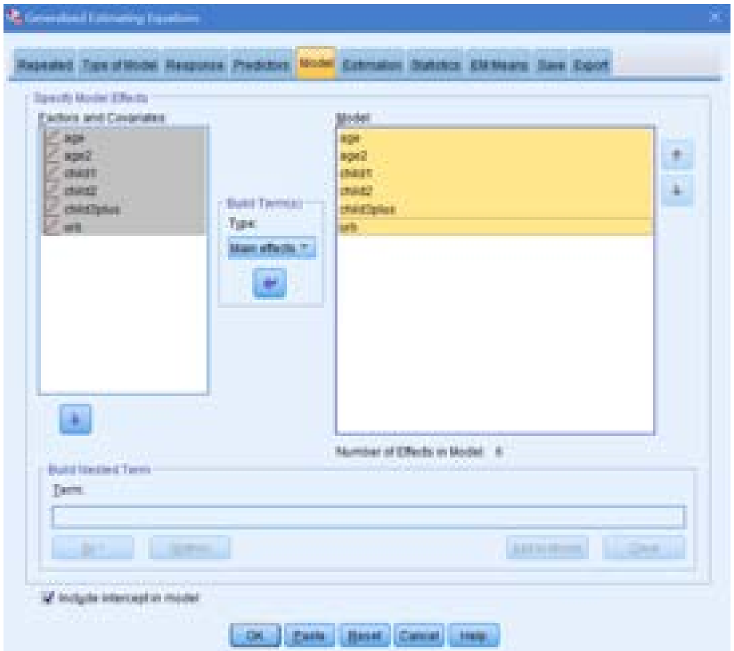
6. Since a logistic regression will be run, users should make sure that the reference group is correctly specified. In this case, the outcome is a 0 or a 1. As we would like to model the 1s in comparison to the 0s, click on **Reference Category…** and select **First (lowest value)**. If this is not done, the model may estimate the likelihood of getting a 0 compared to getting a 1 (and the resulting coefficients may be in the opposite direction as to what was expected). Click **Continue**.



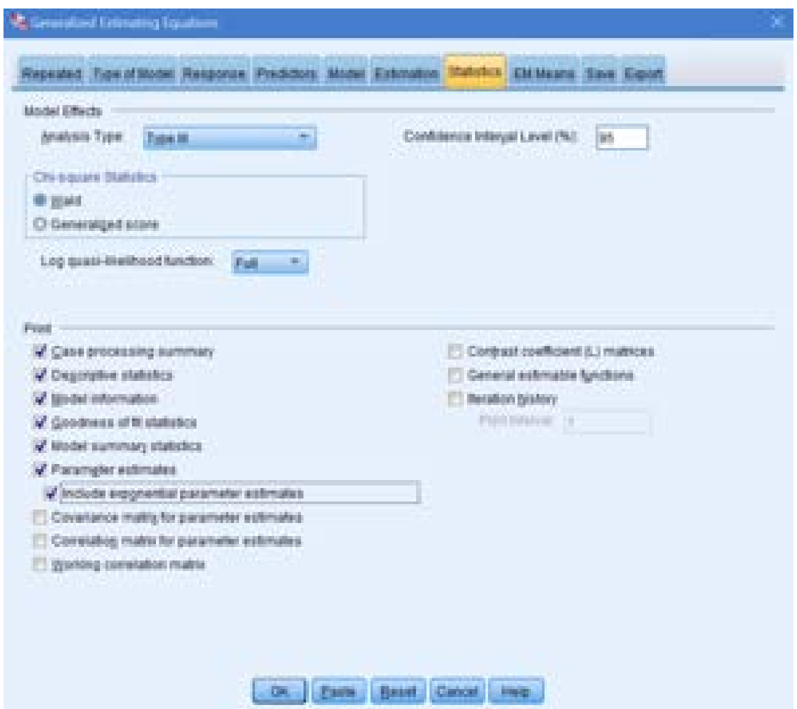
7. Select **Predictors** to include in the model. As all the variables are already numeric (dummy coded as a 1 or a 0), select the variables of interest and place them in the **Covariates:** section. [in this example, `nochild` is not included as this is the reference category]



8. Click on the **Model** tab. Select the variables of interest on the left and click on  to transfer them to the **Model** box on the right.



9. At this point, users can click **OK** to run the model. However, as this is a logistic regression model, it may be easier to interpret the exponentiated log odds which are odds ratios (*OR*s). To output the *OR*s, click on the **Statistics** tab. Click on **Include exponential parameter estimates**. Click **OK**. Do not do this if running a linear model.



10. The following is a portion of the sample output showing the regression coefficients, standard errors, and statistical significance of the estimates—and also the odds ratios under the heading **Exp(B)**.

Parameter Estimates

			95% Wald Confidence Interval		Hypothesis Test				95% Wald Confidence Interval for Exp(B)	
Parameter	B	Std. Error	Lower	Upper	Wald Chi-Square	df	Sig.	Exp(B)	Lower	Upper
(Intercept)	-.985	.2004	-1.378	-.593	24.179	1	.000	.373	.252	.553
age	.004	.0086	-.013	.021	.197	1	.657	1.004	.987	1.021
age2	-.004	.0007	-.006	-.003	41.059	1	.000	.996	.994	.997
child1	.778	.1881	.409	1.147	17.095	1	.000	2.177	1.506	3.148
child2	.868	.1603	.554	1.182	29.319	1	.000	2.383	1.740	3.262
child3plus	.870	.2022	.474	1.266	18.518	1	.000	2.387	1.606	3.548
urb	.644	.1532	.343	.944	17.654	1	.000	1.904	1.410	2.571
(Scale)	1									

Dependent Variable: cuse
Model: (Intercept), age, age2, child1, child2, child3plus, urb

Appendix B: Small sample correction

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2020.06.25

1. Load in the libraries

- sampling is used to facilitate the random selection of 20 groups (J = 20).
- geesmv (Wang et al., 2016) has eight different standard error corrections for GEEs. It is used to estimate the standard error adjustments.
- geepack is still used to estimate the GLM using GEE.

```
library(mlmRev) #has the Hsb82 dataset
library(sampling) #to randomly select 20 schools
library(geesmv) #for small sample correction
library(geepack) #for geeglm

data(Hsb82)
set.seed(123)
sel <- cluster(Hsb82, "school", 20, method = 'srswor')
J20 <- getdata(Hsb82, sel) #create the J20 dataset
J20$school <- droplevels(J20$school) #remove unused school factor levels
length(table(J20$school)) #how many schools

## [1] 20
```

2. Run the model

```
gee1 <- geeglm(mAch ~ sx + minrty + cses + sector + meanses + cses * sector,
id = school, corstr = 'exchangeable', data = J20)
summary(gee1)

##
## Call:
## geeglm(formula = mAch ~ sx + minrty + cses + sector + meanses +
##     cses * sector, data = J20, id = school, corstr = "exchangeable")
##
##  Coefficients:
##                     Estimate Std.err    Wald Pr(>|W|)
## (Intercept)          14.3923  0.8595 280.376  < 2e-16 ***
## sxFemale             -1.8082  0.3549  25.961 3.48e-07 ***
## minrtyYes            -2.0620  0.5752  12.853 0.000337 ***
## cses                  3.1651  0.6432  24.215 8.62e-07 ***
## sectorCatholic        0.9306  1.4555   0.409 0.522568
## meanses               3.3461  1.7885   3.500 0.061353 .
## cses:sectorCatholic  -1.7145  0.6805   6.347 0.011760 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From Huang, F. L. (2021). Analyzing cross-sectionally clustered data using generalized estimating equations. *Journal of Educational and Behavioral Statistics.* doi: 10.3102/10769986211017480

```
##
## Correlation structure = exchangeable
## Estimated Scale Parameters:
##
##             Estimate Std.err
## (Intercept)    33.48   1.604
##   Link = identity
##
## Estimated Correlation Parameters:
##       Estimate Std.err
## alpha  0.07645  0.0246
## Number of clusters:   20  Maximum cluster size: 60
```

3. Compute adjusted standard errors

To get the corrected standard errors, use the `GEE.var.md` function. The specification is the same as with the `geeglm` function. Note though that a difference is that the cluster variable is surrounded in quotes. The `GEE.var.lz` function computes the standard Liang & Zeger (1986) robust standard errors.

```
gee.lz <- GEE.var.lz(mAch ~ sx + minrty + cses + sector  + meanses + cses * s
ector, id = 'school', corstr = 'exchangeable', data = J20)

## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27

## running glm to get initial regression estimate

##       (Intercept)            sxFemale          minrtyYes                 cses
##            14.474              -2.388             -2.053                3.143
##      sectorCatholic             meanses cses:sectorCatholic
##             1.160               3.139             -1.717

gee.md <- GEE.var.md(mAch ~ sx + minrty + cses + sector  + meanses + cses * s
ector, id = 'school', corstr = 'exchangeable', data = J20)

## Beginning Cgee S-function, @(#) geeformula.q 4.13 98/01/27
## running glm to get initial regression estimate

##       (Intercept)            sxFemale          minrtyYes                 cses
##            14.474              -2.388             -2.053                3.143
##      sectorCatholic             meanses cses:sectorCatholic
##             1.160               3.139             -1.717

lz.se <- sqrt(gee.lz$cov.beta) #sqrt to get the SE
md.se <- sqrt(gee.md$cov.beta)
```

A comparison between the two SEs shows that the MD SEs are higher (more conservative) vs. the LZ SEs. MD SEs can be 10 - 20% larger.

```
data.frame(lz = lz.se,
           md = md.se)
```

```
##                          lz     md
## (Intercept)          0.8595 1.0938
## sxFemale             0.3549 0.3784
## minrtyYes            0.5753 0.6620
## cses                 0.6432 0.7698
## sectorCatholic       1.4554 2.1057
## meanses              1.7884 2.6795
## cses:sectorCatholic 0.6805 0.8051
```

To use the adjusted standard errors, need to use the `coeftest` function in the `lmtest` package. The geesmv function only provides the variances/standard errors. To use them, need to place the SEs in a diagonal matrix which is used in the `coeftest` function.

For comparison, compute the Liang and Zeger (LZ; 1986) standard errors and the Mancl and DeRouen (MD) standard errors.

```
library(lmtest)
se1 <- diag(gee.lz$cov.beta)
coeftest(gee1, se1)
```

```
##
## z test of coefficients:
##
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)            14.392      0.860   16.74  < 2e-16 ***
## sxFemale               -1.808      0.355   -5.09  3.5e-07 ***
## minrtyYes              -2.062      0.575   -3.58  0.00034 ***
## cses                    3.165      0.643    4.92  8.6e-07 ***
## sectorCatholic          0.931      1.455    0.64  0.52253
## meanses                 3.346      1.788    1.87  0.06134 .
## cses:sectorCatholic    -1.714      0.681   -2.52  0.01176 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(gee1) #should be the same
```

```
##
## Call:
## geeglm(formula = mAch ~ sx + minrty + cses + sector + meanses +
##     cses * sector, data = J20, id = school, corstr = "exchangeable")
##
##  Coefficients:
##                      Estimate Std.err   Wald Pr(>|W|)
## (Intercept)            14.392   0.860 280.38  < 2e-16 ***
## sxFemale               -1.808   0.355  25.96  3.5e-07 ***
```

```
## minrtyYes            -2.062   0.575  12.85  0.00034 ***
## cses                  3.165   0.643  24.21  8.6e-07 ***
## sectorCatholic        0.931   1.456   0.41  0.52257
## meanses               3.346   1.788   3.50  0.06135 .
## cses:sectorCatholic  -1.714   0.681   6.35  0.01176 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = exchangeable
## Estimated Scale Parameters:
##
##             Estimate Std.err
## (Intercept)     33.5     1.6
##   Link = identity
##
## Estimated Correlation Parameters:
##       Estimate Std.err
## alpha   0.0765  0.0246
## Number of clusters:   20  Maximum cluster size: 60
```

Compare this now to the Mancl and DeRouen (2001) correction (the standard errors are larger):

```
se2 <- diag(gee.md$cov.beta)
coeftest(gee1, se2)
```

```
##
## z test of coefficients:
##
##                     Estimate Std. Error z value Pr(>|z|)
## (Intercept)           14.392      1.094   13.16  < 2e-16 ***
## sxFemale              -1.808      0.378   -4.78  1.8e-06 ***
## minrtyYes             -2.062      0.662   -3.11   0.0018 **
## cses                   3.165      0.770    4.11  3.9e-05 ***
## sectorCatholic         0.931      2.106    0.44   0.6585
## meanses                3.346      2.679    1.25   0.2117
## cses:sectorCatholic   -1.714      0.805   -2.13   0.0332 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

References

Liang, K. Y., & Zeger, S. L. (1986). Longitudinal data analysis using generalized linear models. *Biometrika, 73*, 13-22.

Mancl, L. A., & DeRouen, T. A. (2001). A covariance estimator for GEE with improved small-sample properties. *Biometrics, 57*, 126-134.

Wang, M., Kong, L., Li, Z., & Zhang, L. (2016). Covariance estimators for Generalized Estimating Equations (GEE) in longitudinal analysis with small samples. *Statistics in Medicine, 35*, 1706-1721. https://doi.org/10.1002/sim.6817